The Influence of Euclidean Geometry on Current technologies and Various Applications of Euclidean Geometry

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[Abstract] The Euclidean geometry has been considered as an irrefutable truth for a long time. Along with development of science and technology, however, Euclidean geometry has lost its power. Especially, whether the parallel postulate is true in reality had been questioned by many scholars and non-Euclidean geometry has developed. However, although non-Euclidean geometry has developed and been focused, Euclidean geometry kept influencing modern world in various fields including education and architecture. The unique characteristic of Euclidean geometry lies under the continuousness of its influence. The characteristic that is focused in this report is academic methodology of deductive reasoning with self-evident axioms. 'The Elements' by Euclid has been considered as standard textbook of mathematics education irreplaceably. Despite all the criticisms toward Euclidean geometry being used in proper mathematics curriculum represented by syndrome of "Euclid must go", Euclidean geometry still remains as important part of mathematics curriculum. This was because Euclidean geometric language is able to be understood intuitively and be formalized, which leads to the fact that Euclidean geometry preserve 'meaning' represented by spatial intuition while formally defined. As geometry and architecture heavily influence each other, Euclidean geometry obviously influenced architecture from ancient Greece to modern world. In this report, Euclidean geometry in Parthenon, Satellite city tower, and music hall of Berlin Philharmonic will be focused. At last, this report will focus on the importance of Euclidean geometry while using computer aided design and graphical illustration of structures. Throughout the report it is noticeable that even today Euclidean geometry remains alive and keeps exercising a far-reaching influence.

[keyword] Euclid, Euclidean geometry, methodology, deductive reasoning, axiom, postulate, The Elements, Geometric language, optics, computer aided design, graphical illustration, ground plan

Introduction

We actually face geometry countlessly on a daily basis. All different objects and buildings that we see are based on geometry. Such geometry developed along with ancient civilization and has made much advancement. Emergence of Egyptian's problem-solving geometry lead to practical studies, abstract concepts, and different fields of mathematics that are incorporated in modern times. For example, algebra was subdivided into analytic geometry and algebraic geometry. Euclidean geometry, the main topic of this research paper, established topology based on an axiomatic system and homology. Among other forms of geometry and mathematics, Euclidean geometry provides a special meaning to the modern society and technology. The following is the research about Euclidean geometry and its effects on the society. Euclidean geometry has significance in methodology. If previous studies were based more on discovery rather than proves to set a formula, Euclidean geometry consists of assuming a small set of intuitively appealing experiences and deducing many other theorems from them. It has an abstract characteristic of unfolding the "obvious" and irrefutable axioms into an inevitable conclusion. By using such method, Euclid based and completed his theory of proving mathematics only by using given axioms and definitions.
His theory was not based on newly found studies but rather proofs of how previously set geometric theories were valid and established. In fact, most of the theories Euclid organized were already known to other mathematicians. However, Euclid revealed deductive reasonings behind how those propositions were valid by comprehensive inferences and logics. In short, he was the first to derive axiomatic system and use standardized proofs.

Like already mentioned, Euclid's axioms seemed so intuitively irrefutable that any theorems proved from them were deemed true in an absolute, often metaphysical, sense. However, overtime, people started to doubt their veracity and some even claimed it false. In addition, in the modern society, Euclidean geometry contradictions were more discovered. For example, Albert Einstein's theory of general relativity is that physical space itself is not Euclidean and Euclidean space is a good approximation for it only over short distances (relative to the strength of the gravitational field). Even with this, Euclidean geometry has been considered valid for a long time and it is hard to dispute against it due to its prolonging effects on the modern society.

Thus, in this research paper, Euclidean geometry's characteristics and its limitation will be thoroughly discussed. In addition, this research paper will also examine its effects on future society.

What is Euclidean Geometry?

A. Definition of Euclidean Geometry

Like mentioned earlier, Euclidean geometry is the study of planes and solid figures on the basis of axioms and theorems. Also, it can mean a study of geometry that satisfies the parallel lines axiom.

So, what is the definition of geometry? Geometry is defined as the branch of mathematics concerned with the properties and relations of points, lines, surfaces, solids and higher dimensional analogs. F. Klein, in his influential Erlangen program, defined geometry as a pair \((S, G)\) where \(S\) is a Lie group and \(G\) is a closed Lie subgroup of \(S\) such that the coset space \(S/G\) is called space of the geometry. By following this, Euclidean geometry is the geometry that is set by three-dimensional space and its consolidated conversion groups.

Geometry started as a practical study for ancient agriculture and constructions. Today, history of it can still be found in Babylonian clay tabs, Egyptian papyrus, and ancient books on field arithmetic. Such studies have lists of experimental trial and error data; for example, many studies regarding the findings of pi. What used to be only for experiments started to get proven into consolidated laws by the ancient Greeks. By this, geometry, originally an axiom, turned into a field of studies for its problem-solving characteristics. Especially, Euclid published a summary of ancient Greek mathematical discoveries, which is later used as a textbook. He proved several geometrical summaries by establishing valid postulates and axioms, which later develop into the field of Euclidean geometry.

In the recent years, Descartes implemented rectangular coordinates, which connected geometry to coordinates using algebra. Analytic geometry is defined as a branch of algebra that is used to model geometric objects: points, lines and circles. In the 19th centuries, other mathematicians started doubting the Euclidean geometry and its correlation to parallel axiom; they determined that it can prove not only Euclidean space but also other forms of space. David Hilbert and other mathematicians recreated geometric axioms and published the Hilbert axiom. Both Euclidean and non-Euclidean geometry use other forms of problem proving skills. The important point here is not that Euclidean geometry and non-Euclidean geometry have different methodical research but rather that they use different fundamental theories.

B. “The Elements”
The Elements is mainly a systematization of earlier knowledge of geometry. Euclid deduced 465 summaries from 10 postulates. The ten postulates are all validated by others and the contents are the following.

If $A=B$ and $B=C$, then $A=C$. If $A=B$, then $A+C=B+C$. If $A=B$, then $A-C=A-C$. If all parts are the same, everything is the same because everything includes all parts. Two points determine a line. A segment can elongate into a straight line. A circle has the same radius from any points. All right angles are the same. Through any line and a point not on the line, there is exactly and only one line passing through that point parallel to the line.

In the 19th centuries, many thinkers attempted to use non-rigorous geometrical proofs to prove the last postulate, yet were unsuccessful. It is believed that Euclid might have also doubted it. Proving such is almost impossible because they always used the postulate itself in the proving. So, there was a slight change to the postulate. For example, in the Playfair’s axiom, it is stated that “In a plane, through a point not on a given straight line, at most one line can be drawn that never meets the given line”.

Euclid assumed a small set of intuitively appealing axioms and deduced many other propositions (theorems) from these. Euclid proved the Pythagorean theorem (the square of the hypotenuse is equal to the sum of the squares of the other two sides) like the following.

Let the triangle ABC be a right-angled triangle with BAC. Both angles BAC and BAG are right angles and lines AB and AH are the same as well as CA and AG. Since angle DBC and ABF are right angled, angles ABD and CBF are the same. Line BC and BD are the same so line AB and BF are the same. Thus, triangle CBF and ABD are equivalent. Triangles CBF and ABF have the areas. So, square ABFG has the same area as rectangle BL. Using the same reason, the areas of square ACKH and rectangle CL are the same. Thus, the addition of areas of square ABG and rectangle ACKH equal the area of square BDEC. Finally, AB squared plus AC squared equals BC squared.

C. Evaluation of the value of Euclidean Geometry

Euclidean geometry, like the Bible, is considered the most read book in the world. The axiomatic system is still practically used today. Even the most rationalist philosophers like Thomas Aquinas used methods similar to the axiomatic system. However, Euclidean geometry is no longer considered completely valid like it used to be. Because, in the 19th and 20th centuries, many worldly geometric figures were too complex to be proven purely by the Euclidean methods. What used to be only for the parallel postulate further got expanded to Einstein’s theory of relatively, which significantly modified the view. However, many microscopic analyses of the universe can still be explained by the Euclidean method.
2. Influence of Euclidean Geometry on Education

A. Current state of affairs of Euclidean Geometry on Education

Euclidean geometry, especially the theories, is considered the textbook of modern mathematics. It is still included in elementary, middle school and high school curriculums. The materials are not just included in the Korean educational system but also included in the NCTM’s educational curriculums of other countries in the world. In Greece, there is much more focus on the Euclidean geometry as they believe that it enhances students’ thinking skills. In 1862, Todhunter noted in the preface of Euclidean theories, “In any attempt to set out the essential of Euclidean geometry as strictly logical system, and, in particular, the difficulty of making the best selection of unproved postulates or axioms to form the foundation of the subject.” However, there is still numerous controversies regarding Euclidean education, which seem to get worse over time. There has been an increase in syndromes like “Euclid must go” and many suggested other forms of education other than the Euclidean.

B. Criticisms about Euclidean Geometry on Education

John Perry at “The teaching of Mathematics” lecture in Glasgow stated that he believed that Euclidean geometry put precision at focus excessively, which can be harmful to educational purposes. Moore at a United States Mathematics Forum, “On the foundations of mathematics”, stated that Euclidean geometry subdivides too excessively that it can’t provide a logical sense to others. Other education analysts also doubt the true effects of Euclidean geometry. The following can be summarized into two main categories. First, it means that Euclid’s theories have no practical effects. For examples, when explaining about the characteristics of triangle, Euclidean geometry doesn’t explain thoroughly. Next is the theoretical reason. Hilbert in his theories often mentioned that other forms can provide more popular ideas rather than the Euclidean way.

C. Refutation to the criticism and Influence of Euclidean Geometry to education

First, the fact that Euclidean geometry is not practical is completely false. Although those facts can’t be refuted, however, the positive effects can’t really be proven. Incorporating too much algebra into geometry has simply made students memorize different equations. However, ancient geometric problems can intrigue numerous students by its ability to be solved in many different ways. Having no one answer allows enhancement of thinking skills of students. By invoking problem solving skills, students can not only improve their effort level, and concentration skills. Although Euclidean geometry might not be practical in the modern days, it surely influences the modern educational system and geometrical thinking skills of those students.

Also, it is hard to pinpoint that Euclidean geometry is incomplete and its rigidity is mathematically non-educational. First, there is no direct definition of rigidity and there is no need to make a strict form for students. R. Thom compared everyday language, Euclidean geometric language, and algebraic language in formalization, intuitive understanding, and sentence construction possibilities. The results are the following.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Formalization Possibility</th>
<th>Intuitive Understanding Possibility</th>
<th>Sentence Construction Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Everyday Language</td>
<td>Formalization Impossible</td>
<td>Clear Meanings</td>
<td>Poor</td>
</tr>
<tr>
<td>Euclidean Language</td>
<td>Formalization Possible</td>
<td>Clear Meanings</td>
<td>Good</td>
</tr>
<tr>
<td>Algebraic Language</td>
<td>Formalization Possible</td>
<td>Unclear Meanings/ Meaningless</td>
<td>Good</td>
</tr>
</tbody>
</table>
With this, it can be concluded that Euclidean language is between levels of everyday language and algebraic language. Euclidean language can have clear meanings and can construct sentences yet unable to set formalization exactly. Due to these characteristics, it is hard to incorporate Euclidean language directly into the education system. Too much formalization has no significance in students because summaries are often too complicated and can lead into meaningless prose. In short, there is no need to give abstract language to students because by default, they will not understand. In order to preserve the Euclidean geometry's intuitive meaning, it should be left in the education system.

3. Application of Euclidean Geometry on Art and Architecture

A. Geometry and Architecture

Robin Evans said “Architects do not produce geometry, they consume it. Such at least would be the inevitable conclusion of anyone reviewing the history of architectural theory.” Like this, geometry is an important factor in societal, idea, and technological development. In the “Hyundai Architecture”, it is stated that the societal paradigm and geometry has an intersection, which enhanced the development of architecture. Due to the immense number of Euclidean designed buildings and architectures, modern designs began to develop.

B. Optics of Ancient Greece and Euclidean Geometry

Euclidean optics is also based on summary, and postulates like the previous book. The following is a sector of Euclidean optics.

“Let it be assumed that rectilinear rays proceeding from the eye diverse indefinitely; that the figure contained by a set of visual rays is a cone of which the vertex is at the eye and the base at the surface of the objects seen; that those things are seen upon which visuals rays fall and those things are not seen upon which visual rays do not fall; that things seen under a larger angle appear larger, those under a smaller angle appear smaller, and those under equal appear equal; that things seen by higher visual rays appear higher, and things seen by lower visual rays appear lower; that similarly, things seen by rays further to the right appear further to the right, and things seen by rays further to the left appear further to the left; that things seen under more angles are seen more clearly.”

Euclid assumed that visual arrays follow the rules of reflection and angle, and explained the objects following such rules. These were mirrored into the architecture. Within these postulates, postulate 5 and 6 will be used to explain how they helped the construction of Parthenon. First, postulate 5 states “if two equally sized objects are in different locations, one closer seems larger”. Second, postulate 6 states “When looking at parallel lines from a distance, it doesn’t seem like they have the same distance in between”. With this information, ancient architects believed that if they built a building like Figure E, it will look like F due to optical illusions. So, they rather built it like Figure G to fix those issues.
Although the actual proportions were all different, it had an effect of looking sharp.

C. Examples of Euclidean Geometry applied in Architecture

Mexico City’s Satellite tower and Berlin’s Philharmonic music hall were chosen as representations of Euclidean geometry architecture by the “Uses of Euclidean Geometry on Modern Architecture” research paper. Its formative aspects were especially noted. Formative characteristics are largely divided into two categories: 2D and 3D. Methodical combination include juxtaposition, conjunction and intersection while arrangement methods include linear, corporation, regulation, freedom and radiation. Each methods have no set of rules but rather done freely.

In Mexico City’s Satellite tower, the free arrangement style was installed. It can be shown that it has acute angled triangles. With this, the building has strong sense of direction and speed.

Berlin’s philharmonic music hall has irregular two floor plans. When looking at the floor plan, its symbol of three pentagons are noted. Each pentagon has a different set of focus, which further enhances the irregularity.

D. Computer Aided Design and Euclidean Geometry

Nowadays, non-Euclidean geometry has been animated. Nevertheless, Euclidean geometry has heavily impacted the development of CAD and CAM (computer aided design/ computer aided manufacturing). CAD is the use of computers to aid in the creation, modification, analysis or optimization of a design. Through CAD software, engineers and architects can install 2D vector-based drafting system and 3D solid
and surface modelers to 2D and 3D. By this, production cost and time for design are both reduced, which leads to design cycle.

Jorge R. Dantas in "The End of Euclidean Geometry or Its Alternative Uses in Computer Design" stated that in complex architectural designs using CAD and other computer programs Euclidean methods must be used for graphical representation. In order to structurally understand and calculate, Euclidean geometry expression is a must. As previously mentioned by the 19th century mathematicians and physicists, there are discrepancies between the reality and the Euclidean sense of space. With this, there was a dispute whether or not Euclidean method can bring an accurate sense of "reality" using CAD. Daniela Bertol in "Virtualizing with CAD: An Auto CAD Exploration of Geometric and Architectural Forms" mentioned that although Euclidean geometry can’t perfectly correspond to the real physical state, it can help remodel it through mutual physiological logics.

**Conclusion**

It is true that the use of Euclidean geometry has lessened throughout the years. Non-Euclidean geometry emerged as an opposition of Euclidean method; it seems to replace many Euclidean methods. Non-Euclidean geometry can easily be found in architecture, education, and art. Despite all this, Euclidean geometry still is prevalent and effective in many other ways. The values of Euclidean geometry are systematicity and logics. The axioms used behind those theories are still widely used today.

Due to such values, it is still incorporated in modern education systems. It is believed that the Euclidean geometry improves thinking skills. Yet, its intuitive description, formalism, and impracticality are criticized. However, as seen in this research paper, it is clear that it can still provide some educational purposes. Euclidean geometry has significantly impacted the modern architectural skills, and theories. Euclidean optical science especially impacted the ancient Greek buildings, Mexico City’s satellite city tower, and Berlin’s philharmonic music hall. In addition, programs like the Computer Aided Design, which expedites numerous tasks. It is clear that the Euclidean geometry is not perfect, however, Euclidean language and expressions are still widely used today.

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